EXPLORING STUDENTS' UNDERSTANDING OF ORDINARY DIFFERENTIAL EQUATIONS USING COMPUTER ALGEBRAIC SYSTEM (CAS)

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ABSTRACT
Ordinary differential equations (ODEs) are one of the important topics in engineering mathematics that lead to the understanding of technical concepts among students. This study was conducted to explore the students' understanding of ODEs when they solve ODE questions using a traditional method as well as a computer algebraic system, particularly Maple. Ten engineering technology students from the robotics and the mechatronics departments were selected as the research participants. Semi-structured interviews, observations, and students' written scripts were used for gathering the information regarding the use of Maple for solving the given ODE questions. The findings revealed that Maple could assist students in understanding the ODE lesson, especially in getting rid of tedious calculations as well as producing interactive activities while learning mathematics. Generally, students were able to figure out the relationship between mathematical understanding and real-life engineering applications. Some recommendations are made in order to promote the use of Maple among students, particularly in their final-year project.
Keywords: engineering mathematics, Maple, computer algebraic system, ordinary differential equation, technology

INTRODUCTION

Computer algebraic systems (CASs) have become an essential tool in making the teaching and learning process become more meaningful (Ittigson and Zewe, 2003). This idea has been extended by various researchers through studies that evaluate the benefit of using certain elements of CAS during the classroom process, especially for teaching mathematics. The CAS was introduced to the world of education in 1970 (Godarzi, Aminifar, and Bakhshalizadeh, 2009). Nowadays, there are several commonly used CASs in mathematics classrooms, such as Maple, Mathematica, Mathlab, and Mathcad. The justification of using Maple for this research is its availability as well as its user-friendly criteria. Moreover, Maple is useful for solving complicated mathematical problems in both numerical and symbolic forms. Hence, the integration of Maple in the teaching and learning of engineering mathematics will improve the students' outcomes. Technology has been widely incorporated in the teaching and learning of mathematics by most educators, including Meagher (2005); Kilicman, Hassan, and Husain Said (2010); Lua and Yang (1997); Kovacheva (2007); Godarzi, Aminifar, and Bakhshalizadeh (2009); and Slavit, Cooper, and LoFaro (2002). For instance, Slavit and colleagues (2002) conducted an experimental study with the purpose of identifying the effect of CASs on students' attitude and outcomes by using simulations and an instructional approach for teaching differential equations. The findings of their study revealed that students can conceptualize and analyze the ideas that lead to the solutions of differential equations. Godarzi and colleagues (2009) conducted another experimental study in order to determine the impact of Maple 12 on the teaching and learning of the double integral for 44 first-year students, focusing mainly on their procedural and conceptual understanding. The result revealed that the experimental group performed better in terms of conceptual and procedural understanding than the control group. As mentioned by Godarzi and colleagues (2009), conceptual and procedural understanding can be provided through activities using Maple. Conceptualization and computational representation methods are being introduced to students for solving various mathematics problems. Conceptual understanding can be developed through procedural understanding. A learning goal can be achieved if Maple is properly utilized in mathematics classrooms in order to ensure that students can grasp the desired concepts. This leads to an improvement in their attitude towards mathematics; gradually, they can develop the required understanding through the learning process (Kwon *et al.*, 2005).

Although there are many types of CASs in the field of education, Maple is considered one of the most powerful and is commonly used for teaching advanced mathematical topics such as calculus, linear algebra, vector computations, complex numbers, statistics, combinatorics, and number theory (Godarzi, Aminifar, and Bakhshalizadeh, 2009). As compared to the traditional method, Maple has made complicated topics such as multiple integrals easier to comprehend. As such, Maple is used for teaching advanced topics like multivariate calculus (Suanmali, 2008) and set operations (Kilicman *et al.*, 2010). The facilities that Maple provides, such as solving differentiation and integration step by step, plotting two and three-dimensional graphs, and animation features, enable the students to have a better learning engagement in a mathematics class. With all these features, Maple supports the students' understanding of ODE better than the traditional approach.

It is also widely known that mathematics and engineering complement each other in developing mutual understanding among students in the respective fields. Therefore, engineering mathematics has been taught to engineering students in order to provide them with mathematical knowledge and the ability to effectively use computational skills in engineering problems. Through a discussion with some mathematics lecturers at the Universiti Kuala Lumpur Malaysia France Institute, the researcher found out that tedious calculation, lack of basic understanding in calculus, and problems in visualizing the solution of mathematical questions (Godarzi, Aminifar, and Bakhshalizadeh, 2009) were among the common difficulties that students faced while learning ODEs. Hence, this study was conducted to investigate the students' understanding of ODEs when they use Maple.

MATERIALS AND METHODS

The research participants were ten engineering technology students who enrolled in an industrial automation and robotic program as well as a mechatronics program. The selection was based on the assessment for first-order ODEs; the assessment results were grouped into three categories: the lowest, the average, and the highest test score. Four students scored the lowest marks, three students scored average, and three students scored the highest marks. Only ten students were chosen because of the limited number of computers with Maple 13 in the laboratory as well as the willingness to participate in this study. These students have to enroll in the engineering mathematics course as it is a prerequisite for the control systems and robotics subjects. The engineering mathematics course consists of topics like differential equations, Laplace transforms, and Fourier series and is conducted three hours per week throughout the fourteen weeks per semester. As such, ODE is a fundamental concept in engineering modeling, including vibration and electrical circuits (Lawson 1997). Additionally, the high number of failures in every semester has become the justification to introduce Maple 13 in engineering mathematics.

The data was collected from three different sources, including observations, interviews, and analysis of students' written scripts. Three phases of the study were conducted by the researcher in order to ensure the research process was implemented successfully. The lecturer who participated in this study was a female lecturer with more than 15 years of teaching experience in the field of mathematics, especially at the diploma and degree levels. Her teaching style oriented more towards the traditional approach, and most of the tutorial sessions as well as the lectures were dominated by the lecturer. Since all the students were required to attend the lectures as well as the tutorial sessions, the activity was the same for all students in the first phase of the study. A quiz on first-order ODEs was given during the first phase of the study. The students' scores were ranked: three students got the highest marks, three average marks, and the remaining four students the lowest marks for the quiz. Their written scripts were analyzed, and the researcher interviewed them in order to get more detailed information. During the next session, the ten students were called to attend a Maple 13 training session that was conducted by the lecturer. The session lasted two hours, and the students were told to explore the software by themselves after the session. The third session was conducted with the intention of comparing students using an algebraic solution and students using Maple 13. The students were observed throughout the three phases, and we focused especially on the students' interest in using Maple during the mathematics class as well as the lecturer's readiness in integrating Maple in her teaching.

RESULTS

During the lecture on differential equations, which covers first-order ODEs, the students posed some questions to the lecturer. However, the questions revolved around basic integration that needs to be used for solving ODEs. When this happened, the lecturer gradually started to introduce several types of first-order ODEs with the intention that the students could distinguish among the different types of ODEs. The tutorial session was fully occupied since most students took the chance to ask questions as well as try tutorial questions. At the end of the first-order-ODE lesson, a quiz was given in order to test their understanding of the topic before proceeding to the second-order ODEs. The ten students were selected on the basis of their ranking on the given quiz. The students' written answers gave an idea to the lecturer regarding what should be included in the next lesson on second-order ODEs.

The second-order ODE was introduced by starting the lecture with the applications of ODEs in engineering applications such as vibration systems and circuit problems (Lawson 1997) so that the learning of second-order

ODEs becomes more meaningful. The tutorial session for the second-order ODEs was conducted upon the completion of each sub-topic related to second-order ODEs, including homogeneous ODEs and non-homogeneous ODEs. At this stage, more problems, especially those related to the specific method of performing integration, were encountered. The weak and average students were found to have problems in recognizing the suitable method for solving the integration. They were not able to complete the whole working solutions for the given questions. However, the good students seemed to enjoy the challenge as the questions became progressively more difficult. When they were given the application questions of ODEs in engineering problems, one of the good students was able to get the correct solution.

Next, all the written scripts were analyzed in order to identify the problems among the research participants in solving second-order ODEs. Three types of questions for this topic were chosen: one on homogeneous second-order ODEs, another on non-homogeneous ODEs, , (as shown in Table 1), and the other on the application.

Table 1: Types of second-order ODE questions

Type	Question
Homogenous	$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=0$
Non-homogeneous	$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=e^t$ with the given initial condition; $t=0$, $x=1$ and $\frac{dx}{dt}=\frac{3}{2}$

Going through the scripts, the researcher could draw a conclusion on the students' problems in understanding the lesson. Most students, except the good students, faced difficulties in producing the auxiliary equations such as $m^2+6m+5=0$ as well as writing a general solution and particular solutions. Using the given initial value problem (IVP) also created confusion among the students in producing a particular solution. To clarify the confusion, the next session on Maple training was conducted at the end of the lesson on ODEs.

The Maple session was handled by the lecturer since she had undergone Maple training before. She used a Maple worksheet for solving questions on integration and differentiation. The lecturer briefed the students on how to use certain features of Maple, such as menus, icons, and shortcut commands. Basic calculations on some commonly used expressions in mathematics can be carried out using the shortcut menus, as shown in Figure 1. For the session, a step-by-step approach was used in order to ensure that the students could follow the instructions. The lecturer has also mentioned the do's and don'ts of exploring Maple.

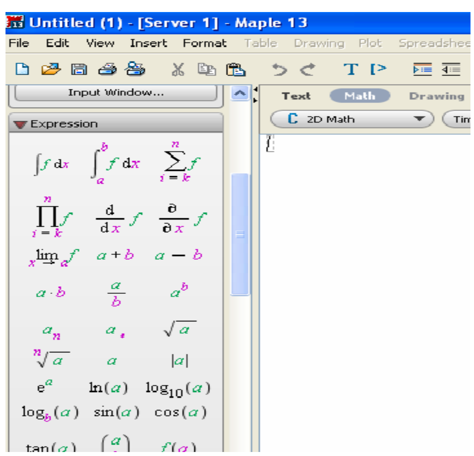

Figure 1. Some commonly used mathematical expressions in Maple 13.

Using similar types of questions as those given in Table 1, the lecturer requested the students to type the Maple commands that were shown on the whiteboard, which was projected by the LCD. Students were reminded to avoid mistakes, especially in the spelling of commands as well as spacing, in order to produce the same and correct answers. The following questions were given during the session:

```
> with(inttrans)
[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier,
  invhilbert, invlaplace, invmellin, laplace, mellin, savetable]
```

Example 1

$with(plots):$

$ode := diff(x(t), t, t) + 6\, diff(x(t), t) + 5\, x(t) = 0$

$$\frac{d^2}{dt^2} x(t) + 6 \left(\frac{d}{dt} x(t) \right) + 5\, x(t) = 0$$

$dsolve(ode)$

$$x(t) = \_C1\, e^{-5\,t} + \_C2\, e^{-t}$$

Example 2

$ode1 := diff(x(t), t, t) + 6\, diff(x(t), t) + 5\, x(t) = 2 + 3\,t$

$$\frac{d^2}{dt^2} x(t) + 6 \left(\frac{d}{dt} x(t) \right) + 5\, x(t) = 2 + 3\,t$$

$dsolve(ode1)$

$$x(t) = e^{-5\,t}\, \_C2 + e^{-t}\, \_C1 - \frac{8}{25} + \frac{3}{5}\,t$$

Next, the lecturer let them solve ODEs for a particular solution, as shown in the next part. The justification of giving this question to them was to show the difference between the general solution and the particular solution of an ODE question. During this session, the students seemed to be enjoying the lesson. While doing the activity, the students were asked to compare their answers from the written scripts and the answer produced by Maple; all the comments were recorded by the researcher. Some of them started to try with different questions other than the chosen questions. They even asked the lecturer on how to get the graphical representation of the obtained function.

Determine the particular solution of ode1 from line (5)

$ics := (x(0) = 0, D(x)(0) = 1)$

$$x(0) = 0, D(x)(0) = 1$$

Example 3

$dsolve(\{ode1, ics\})$

$$x(t) = -\frac{9}{50}\, e^{-5\,t} + \frac{1}{2}\, e^{-t} - \frac{8}{25} + \frac{3}{5}\,t$$

The application question was discussed in the last session, and the lecturer recommended them to surf the information on engineering applications at Maplesoft Application Center, which can be found at http://www.maplesoft.com. The applications cover a variety of scopes in the field of engineering, as shown in Figure 2.

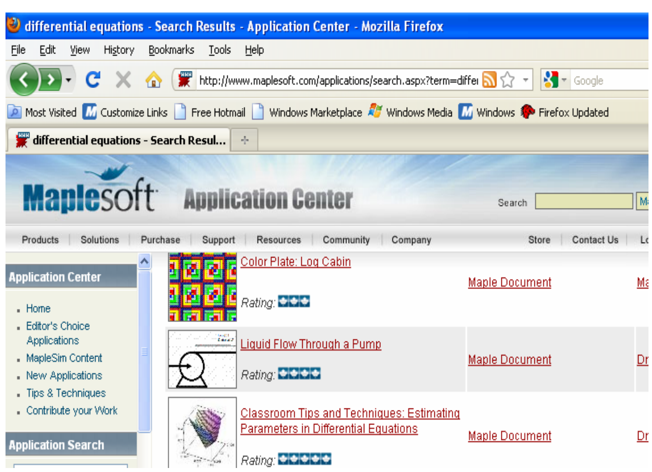

Figure 2. Screenshot of the Maplesoft Application Center.

DISCUSSION

The discussion is divided into three parts, which consist of the research participants, the lecturer, and Maple. The interconnection of all the components of the research will reflect their importance in making the teaching and learning process of ODEs a success. Firstly, most of the students are found to lack the basic understanding of calculus, which is considered a prerequisite for ODEs. Based on the given responses during the interview, the students face difficulties in remembering the methods learnt in integration and differentiation. This is consistent with what has been reported by Kwon, Allen, and Rasmussen (2005). They have a better understanding of procedural knowledge than of conceptual knowledge. The students are able to produce step-by-step explanation but are unable to provide justification. The learning process has been transformed from a directed approach to a non-directed approach by the introduction of the Maple training session. Most of them also found that using Maple was useful to them, especially in reducing the burden of having a series of long working steps. Since most of the ODE questions involved several steps in getting to the final solution, the use of Maple has provided them a better understanding of certain fundamental concepts like general solutions, complementary functions, and particular integrals. The additional knowledge was because of the graphical representation, which was not fully discussed during the tutorial. They also emphasized their preference in using Maple for the next mathematics class. Some comments for this preference include

"I think Maple should be used regularly in mathematics class."
"With Maple, I am able to solve all kinds of mathematics questions, probably…"
"Some special menus of Maple help me to understand integration better..."
"No more complicated working steps in order to solve mathematics questions."

Throughout the training session, the students were intentionally being guided to explore Maple by themselves. The students found that they were given adequate information on how to use Maple for solving questions related to integration, differentiation, and differential equations. With Maple, students managed to identify the appropriate method that can be used for solving any given mathematics question. Most of these students have problems in distinguishing the commonly used terms in ODEs, such as order, degree, homogeneous, non-homogeneous, auxiliary equation, complementary function, and particular integral. These terms are indirectly ignored by these students while the training is in progress because of some of the alternative features offered by Maple. These features include user-friendly criteria, graphical representations, and symbolic and algebraic manipulation.

Further, the lecturer's point of view in integrating Maple in her teaching and learning process has made the task easier than conducting the normal classroom process. Despite the fact that the students have a problem in basic calculus, the lecturer manages to build their confidence in solving ODEs using Maple. Although the lecturer has not fully mastered Maple, but the integration of Maple helps the students in making the learning of ODEs become more meaningful. She has to try out some of the questions using Maple before explaining them to the students. She seemed ready to integrate Maple in her teaching, but the conceptual understanding of ODEs is still a necessity for them. Maple can act as a tool, but the main aim would still be the procedural and conceptual understanding. Using Maple for teaching mathematics has created confusion to the lecturer on how to deliver the mathematical knowledge to the students (Aydin, 2005). The advantage of using Maple in her teaching is that it would enable the students to collaborate with other students for completing the given tasks. The communication changes to a two-way interaction of the lecturer and the students. The class becomes livelier because of the participation of all the students in exploring Maple than what the teacher expected. On the other hand, the lecturer should be aware that she has to make some modifications to her teaching because of some of the changes made during the classroom process (Schneider, 1999). They have to make sure that the students know what to do when Maple fails to provide the expected answer (Kovacheva, 2007).

Maple plays a number of important roles in ensuring a smooth teaching and learning process. One of these roles is that of a didactic resource for teaching mathematics (Galan Garcia *et al.*, 2005), particularly in relation to the conceptual understanding of differential equations and the real-life applications, which are mainly discussed in engineering. Clements (1997) has made a list of the roles played by Maple in the field of engineering, such as providing assistance in solving any related mathematics questions and enabling students to concentrate on concepts. Furthermore, Maple should be able to enhance students' learning in making connections between mathematics and engineering as well as sustaining the students' interest in mathematics. The graphical representation and some animation features in Maple should be able to fascinate the students (Buchanan 2003).

CONCLUSION

Although the outcome of this study shows that students are able to benefit from Maple with respect to solving ODEs, but the limitations should be taken into account. The lecturer should be aware that the relationship between the conceptual understanding and the techniques used in Maple would not be properly delivered to the students (Galan Garcia *et al.*, 2005). There is a possibility that a high student interaction with computers may result in a low student–lecturer interaction, which also has a subsequent effect on the process of socialization provided by the learning environment (Aydin, 2005). Moreover, the learning process can be passive; probably, it would encourage undesirable outcomes among students. They may have different interpretations during the process of exploring Maple for solving ODEs, particularly, because of the graphical representations. This is because of the role of computers in providing the students the complete answer while ignoring the process and the insights of the topic (Aydin, 2005). However, the integration of Maple in the teaching and learning of engineering mathematics, particularly for topics such as differential equations, would enhance students' understanding, increase interest in mathematics, and develop their creative thinking. The learning experience becomes more meaningful for students since Maple creates the opportunity for them to explore, investigate, and draw conclusions (Lua and Yang, 1997). Tedious and complex calculations can be done in minimum time, which enables the students to interact with other students as well as their lecturer. The unnecessary calculations can be neglected, and the students have the opportunity to understand important concepts while solving ODEs (Aydin, 2005). It can be concluded that there are many advantages as well as disadvantages of integrating CAS in a mathematics classroom. In order to be effective in using Maple in the mathematics class, the lecturers have to be well-trained in using Maple. More open training sessions on Maple should be offered to all students and lecturers in order to promote the use of Maple in education, mainly the engineering technology education. Thus, the integration of Maple in mathematics would lead to a new direction in making the learning of mathematics more meaningful and enjoyable.

REFERENCES

Aydin, E. (2005). The use of computers in mathematics education: A paradigm shift from computer-assisted instruction towards student programming. *The Turkish Online J. of Educ. Tech.*, 4(2): 27-34.

Buchanan, M. (2003). In Jenny Way & Toni Beardon. *ICT and Primary Mathematics*. Open University Press, Maidenhead, Philadelphia. Challenges of teaching with technology across the curriculum: Issues and solutions. Hershey: Information

Clements, R. (1997). Computer algebra for engineers: The Maple alternatives. *Eng. Science and Educ. J.*, 233-238.

Galan Garcia, J.L., Galan Garcia, M.A., Galvez Galiano, A., Jimenez Prieto, A.J., Padilla Dominnguez, Y., & Rodriguez Cielos, P. (2005). Computer algebraic system: A basic tool for teaching mathematics in engineering. *Proceedings of the International Conference on Multimedia and ICT in Education*, Lisbon, Portugal.

Godarzi, S.Q., Aminifar, E., & Bakhshalizadeh, S. (2009). The impact of using computer algebraic system (CAS) in teaching and learning of "double integral". *Proceedings of the Third International Conference on Science and Mathematics Education (CoSMEd)*, Penang, Malaysia.

Ittigson, R.J., & Zewe, J.G. (2003). *Technology in the mathematics classroom*. In Tomei, L.A. (Ed.). Science Publishing, 114-133.

Kilicman, A., Hassan, M.A., & Husain Said, S.K. (2010). Teaching and learning using mathematics software "The New Challenge". *Proc. Social and Behavioral Sc.*, 8: 613-619.

Kovacheva, T. (2007). Use of the Maple system in math tuition at Universities. *Inter. J. Info. Tech. and Knowledge*, 1: 363-368.

Kwon, O.N., Allen, K., & Rasmussen, C.(2005). Students' retention of mathematical knowledge and skills in differential equations. *Sch. Sc. and Math.*, 105(5): 227-239.

Lawson, D. (1997). The challenge of computer algebra to engineering mathematics. *Eng. Sc. Educ. J.* Dec, 228-232.

Lua, S.K., & Yang, W.C. (1997). Creating teaching and learning material in engineering mathematics using scientific workplace with external program link to Maple. [Retrieved Feb 23, 2010.]

Meagher, M. (2005). The processes of learning in a computer algebra system (CAS) environment for college students learning calculus. Doctoral Dissertation, The Ohio State University.

Slavit, D., Cooper, K., & LoFaro, T. (2002). Understanding of solution to differential equations through context, web-based simulations, and student discussion. School Science and Mathematics, December; *ProQuest Educ. J.*, 380-390.

Suanmali, S. (2008). Maple in mathematics. *Proceedings of the Fifth International Conference on Information Technology: New Generation*. IEEE Computer Society, 528-533.